

## Analysis and Differential Equations

### Individual Overall

Please solve the following problems.

1. Construct real numbers through decimal numbers. Prove that a bounded increasing sequence of decimal numbers has a limit of decimal number.

2. Let  $B_1(0)$  be the unit ball in  $\mathbb{R}^n$  centered at the origin. Assume that the function  $f \in C^2(B_1(0))$ . Prove that

1) If  $f$  satisfies

$$\sum_{i,j=1}^n x_i x_j \frac{\partial^2 f}{\partial x_i \partial x_j} = 0$$

on  $B_1(0)$ , and  $\nabla f(0) = 0$ , then  $f$  is constant in  $B_1(0)$ .

2) If  $f$  satisfies

$$x_i \frac{\partial f}{\partial x_j} - x_j \frac{\partial f}{\partial x_i} = 0, i, j = 1, \dots, n$$

on  $B_1(0)$ , then  $f$  is constant on the sphere  $\{x : x \in B_1(0), |x| = 1/2\}$ .